Process Reliability Analysis

This paper presents a unified approach for obtaining system reliability and availability parameters from block diagram representations of process systems. It starts with computer algorithms for obtaining the minimal paths in a system, then shows how this information can be combined with unit reliability data to obtain the overall system reliability, and the response of system reliability to changes in unit reliability (sensitivities). It then demonstrates how these sensitivities can be used to optimize the system reliability and availability by equipment redundancy. Extension of the methodology to the automated production of system fault trees as well as the mathematical relationship between fault trees and block diagrams are noted.

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SCOPE

The trend to larger, more complex, and highly interdependent process systems has been in evidence for some time. With increasing scale, repair becomes more costly in both time and money, startup and shutdown losses increase, and the vulnerability of the plant becomes a matter of grave concern. Further factors spurring the process industries' interest in reliability analysis are the high cost of maintenance, which now is as much as 10% of plant cost/annum, and the problems associated with insurance, siting, environmental degradation due to failures, and the establishment of a good public image.

In this paper we focus attention on analytical methods whereby system reliability parameters can be obtained from block-diagrammatic representation of process flow sheets. Although other techniques, such as state enumeration methods are treated, primary emphasis is on the development of path enumeration methods by which the reliability parameters of complex (nonseries-parallel) systems can be computed. As demonstrated in the paper, this approach is particularly powerful because it also permits calculation of the sensitivity of the overall system reliability to changes in configuration and unit reliability. These sensitivities, being gradients, are then used in conjunction with integer, gradient-based, optimization algorithms to solve problems involving allocation of equipment and maintenance intervals.

Also discussed are the construction of fault trees and some potentially useful algorithms for their automated construction from block diagrams.

CONCLUSION AND SIGNIFICANCE

The increasing interest of the chemical industry in reliability analysis is clearly demonstrated by the fact that only three articles on this subject appeared in the chemical engineering literature prior to 1971, and more than 16 have been published since. To date, the process industry has depended almost completely on computational methods developed by the military, electronic, aerospace, and nuclear industries where primary concern is catastrophic failure of essentially nonrepairable systems.

In attempting to apply previously developed methods to process systems, one finds that they do not readily lend themselves to many of the problems of interest to chemical engineers who generally must deal with multicomponent streams, and multiproduct plants which have some storage capability and can usually be kept operating under partial equipment, and sometimes complete equipment failure.

In this larger context the results achieved in this investigation only represent a beginning approach to a universal solution to the problem of developing computational methods for analyzing process systems from an availability and hazards standpoint. The primary significance of our research is that it demonstrates that the overall plant reliability can be calculated directly from process flow sheets, that the optimization problem is very complex (and perhaps not even worth attempting for large systems), and that extension of the basic methodology to problems of maintenance scheduling and availability computation is difficult but feasible.

The earliest paper on the application of reliability principles to chemical engineering systems appears to be that of Rudd (1962). Since then there have been a number of others; taken in approximate chronological order these are:

Fan et al. (1967), A Reliability Optimization, study of a desalination unit

Buffham et al. (1971), A Tutorial Review of Process Applications

McFatter (1972), Reliability Experiences in a Large Refinery

Ufford (1972), Equipment Reliability Analysis for Large Plants

Coulter and Morello (1972), Reliability Improvement in Process Plants

King and Rudd (1972), A Reliability Analysis of a Heavy Water Plant

Cox (1972), Analysis of Single-Train vs. Multi-Train Units

Lawley (1973), A Fault Tree Analysis of a Crystallizer

Gaddy and Culbertson (1973), Plant Availability Calculations by Stochastic Programming

Ross (1973), Intermediate Storage for Reliability Im-

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Powers and Tompkins (1973), Fault Tree Synthesis

Allen and Pearson (1973), Optimum Reliability Investment

Gandhi, Inoue, and Henley (1973), Optimization of Process Reliability

Gandhi and Henley (1974), Computer Aided Design for Optimum Systems Availability

Inoue, Gandhi and Henley (1974), Optimal Reliability Design of Process Systems

Rosen and Henley (1974), Optimization of Intermediate Storage

Wood et al. (1974), Process Plant Reliability Optimization by Linear Programming

Freeman and Gaddy (1974), Quantitative Overdesign to Availability Targets

The chronological ordering clearly demonstrates the increasing importance of reliability considerations in chemical process and equipment design. Responsible for this, in part, is the high cost of unreliability in large, highly integrated, complex plants, and the increasing statutory need to quantify hazards related to equipment failure.

In this paper we develop a unified method of using block diagram representations of process systems, for computing and optimizing system reliabilities, and constructing fault trees.

BASIC DEFINITIONS

The most widely accepted definition for reliability is "the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered" (Green and Bourne, 1972).

The mathematical derivation of the reliability function is treated in most reliability texts (Bazovsky, 1961; Gnedenko et al., 1969). Here the general formula is given without proof

$$R(t) = \exp\left(-\int_0^t z(\tau)d\tau\right) \qquad (1)$$

where the function $z(t)$ is called the *failure rate function, hazard function,* or *hazard rate.*

If the system has a failure time distribution with density function $f(t)$, then the failure time distribution function is given by

$$F(t) = \int_0^t f(\tau)d\tau \qquad (2)$$

and

$$R(t) = 1 - F(t) = \exp\left(-\int_0^t z(\tau)d\tau\right) \qquad (3)$$

Typical failure time density functions with their corresponding reliability and failure rate functions can be found in all standard reference texts.

Two other definitions can be usefully introduced at this point; mean time between failures MTBF and availability A.

$$\text{MTBF} = \int_0^\infty R(t)dt \qquad (4)$$

$$A = \frac{\text{MTBF}}{\text{MTBF} + \text{MDT}} \qquad (5)$$

where MDT is the mean downtime.

SYSTEM REPRESENTATIONS

For most process applications, a reliability graph can be considered as consisting of nodes, branches, and modules with directed arrows between them. Figure 1, for example, represents a nonseries, parallel system. In the module representation of reliability graphs more than one module between any two arbitrary nodes is permitted. A module is defined to be a single unit or simply connected parallel units. A configuration permitted as a module is limited to either of the following:

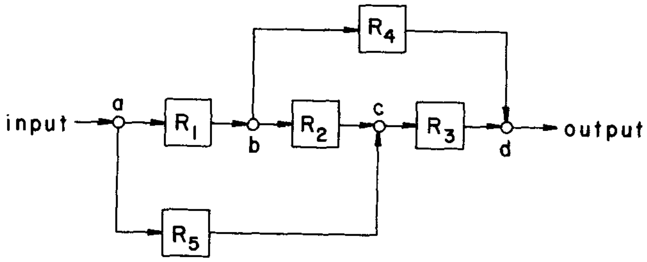

Fig. 1. Reliability graph—non series—parallel system.

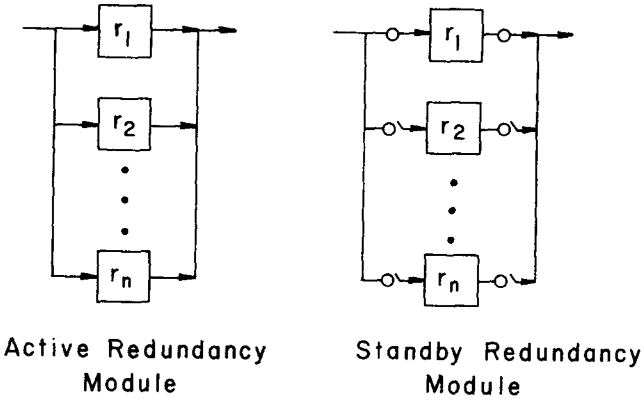

Fig. 2. Multi-unit module.

1. Single Unit Module. A single unit module is one which consists of a single unit. The module reliability is the reliability of the unit itself.

2. Multi-Unit Module. A module which consists of multiple parallel units may be either an active redundancy module or a standby redundancy module as shown in Figure 2. It is possible to distinguish between three types of standby redundancy modules. A unit is called a *cold standby* when the failure probability in standby is zero. When the failure probability is the same in standby as in the service, it is called a *hot standby.* Cases that lie in between these two extremes are called *warm standby.*

The expressions for the module reliability in terms of the type of standby redundance are:

Hot standby (Shooman, 1968):

$$R_i(r_{ij}) = 1 - \prod_{j=1}^{N_i} (1 - r_{ij}) \qquad (6)$$

Cold standby (Benning, 1967):

$$R_i(N_i) = \sum_{j=0}^{N_i - 1} \frac{r_i}{j!}(-\ln r_i)^j \qquad (7)$$

Warm standby (Polovko, 1968):

$$R_i(N_i) = r^0 \left\{ 1 + \sum_{k=1}^{N_i - 1} \frac{Ak}{k!}(1 - r')^k \right\} \qquad (8)$$

where

$$A_k = \prod_{j=0}^{k=1} \left[j + \frac{\ln r^0}{\ln r'} \right]$$

r^0 = reliability of a unit in service, r' = reliability of a unit in standby.

For the hot standby case we have allowed for different units in the same module (r_{ij} = the jth unit in the ith module). In the other cases, all N units in the module are identical. Module reliability expressions for active redundancy and hot standby redundancy are identical.

OBTAINING SYSTEMS RELIABILITY

State Enumeration or Event-Space Method

In this method a list of all possible mutually exclusive states of the system is made, a state being defined by listing the successful and failed elements in the system. In general, the total number of states is 2^n, where n is the number of units or elements in the system. Next, the states which result in successful system operation are identified and the probability of occurrence of each successful state is computed. Finally, all the successful state probabilities which give the system reliability are summed. Although the method is easily programmed (Brown, 1971), it is computationally not feasible for systems having a large number of elements.

Network Reduction Method

In this method the series, parallel and series-parallel subsystems are combined until a nonseries parallel system which cannot be further reduced is obtained. Moscowitz (1958) then suggests the use of a factoring theorem. A particular element x is selected, and the two networks are obtained and generated when x is replaced by a short circuit (perfect connection) and an open circuit. If these two networks are simple series-parallel, they can be reduced. Otherwise the next block y must be selected and the procedure repeated.

Suppose block x is selected and the two networks are generated. The system reliability is then

$$R = R_x \star R|_{R_x=1} + (1 - R_x) \star R|_{R_x=0} \tag{9}$$

where R_x is the reliability of block x. The method is discussed by Banerjee and Rajamani (1972), Buzacott (1967, 1970), and Misra (1972).

Path Enumeration Methods

These methods can be used to determine the reliability of any system not containing dependent failures (Chung, 1971; Kim et al., 1972; Shooman, 1968). A path is a set of elements which form a connection between input and output when traversed in a stated direction. A minimal path is one in which no node is traversed more than once in tracing the path.

Let P_i ($i = 1, 2, \ldots M$) be the ith minimal path in the system. If any path is operable, the system performs adequately. Thus, the system reliability is

$$R = P_r \left\{ \bigcup_{i=1}^{M} P_i \right\} \tag{10}$$

where $\cup$ denotes the union.

By use of the expansion rule for the probability of the union of M events (Feller, 1957), we have the formula

$$R = \sum_{i=1}^{M} P_r\{P_i\} - \sum_{i=1}^{M}\sum_{j>i}^{M} P_r\{P_i \cap P_j\} + \sum_{i=1}^{M}\sum_{j>i}^{M}\sum_{k>j}^{M} P_r\{P_i \cap P_j \cap P_k\} + \ldots + (-1)^{M-1} P_r\left\{\bigcap_{i=1}^{M} P_i\right\} \tag{11}$$

where $\cap$ denotes the intersection.

A cut is defined as a set of elements which if they fail will cause the system to fail regardless of the condition of the other elements in the system. A minimal cut is one in which there is no proper subset of elements whose failure alone will cause the system to fail.

Let C_j ($j = 1, 2, \ldots M$) be the minimal cuts in the system. If any cut is inoperable, the system fails. The system reliability is thus

$$R = 1 - P_r\left\{\bigcup_{j=1}^{M} \overline{C}_j\right\} \tag{12}$$

$$= 1 - \sum_{j=1}^{M} P_r\{\overline{C}_j\} + \sum_{j=1}^{M}\sum_{k>j}^{M} P_r\{\overline{C}_j \cap \overline{C}_k\} - \sum_{j=1}^{M}\sum_{k>j}^{M}\sum_{1>k}^{M} P_r\{\overline{C}_j \cap \overline{C}_k \cap \overline{C}_1\} + \ldots + (-1)^M P_r\left\{\bigcap_{i=1}^{M} \overline{C}_j\right\} \tag{13}$$

where $\overline{C}_j$ denotes the complement of the event C_j; that is, $\overline{C}_j$ denotes the failure of all elements of the cut C_j.

If P_i ($i = 1, 2, \ldots M$) denotes the minimal paths in the system, the system reliability R is as shown in Equations (10) and (11), and the system reliability R in terms of module reliabilities is

$$R = \sum_{i=1}^{M} \prod_{l \in P_i} R_l - \sum_{i=1}^{M}\sum_{j>i}^{M} \prod_{l \in P_i \cup P_j} R_l + \sum_{i=1}^{M}\sum_{j>i}^{M}\sum_{k>j}^{M} \prod_{l \in P_i \cup P_j \cup P_k} R_l + \ldots + (-1)^{M-1} \prod_{l \in \bigcup_{i=1}^{M} P_i} R_l \tag{14}$$

where the members of the ith path, the union of the ith and the jth paths, etc., are denoted by $l \in P_i$, $l \in P_i \cup P_j$, etc.

The total number of terms Z involved in Equation (14) is given by

$$Z = 2^M - 1 \tag{15}$$

SENSITIVITY CALCULATION

It is important that the system reliability expression given by Equation (14) is a bilinear function of each module's reliability. By use of this property, the sensitivity of the system reliability to the module reliability R_i can be obtained by the simple rule

$$S_{R_i} = \partial R/\partial R_i = R|_{R_i=1} - R|_{R_i=0} \quad i = 1, 2, \ldots N \tag{16}$$

where N is the total number of modules in the system. S_{R_i} is a module sensitivity.

Two types of sensitivities are of interest. The first is the sensitivity of the system reliability to a unit reliability. A module sensitivity is itself of this type if the module is a single-unit module. For a multiple-unit module, the sensitivity is given by

$$S_{r_{ij}} = \frac{\partial R}{\partial r_{ij}} = \frac{\partial R}{\partial R_i} \star \frac{\partial R_i}{\partial r_{ij}} \tag{17}$$

where r_{ij} is the reliability of the jth unit of the ith module. For an active redundancy module with different units, one obtains the following expression:

$$S_{r_{ij}} = S_{R_i}\,[R_i|_{r_{ij}=1} - R_i|_{r_{ij}=0}] = S_{R_i} \prod_{\substack{j=1\\ j\neq i}}^{N_i} (1 - r_{ij}) \tag{18}$$

where N_i is the number of units in the ith module. The second type of sensitivity is the sensitivity of the system reliability to the number of units in a module. This type of sensitivity is defined for the active redundancy module or the standby redundancy module with identical units.

$$S_{N_i} = \frac{\partial R}{\partial N_i} = \frac{\partial R}{\partial R_i} \star \frac{\partial R_i}{\partial N_i} = S_{R_i} \star \frac{\partial R_i}{\partial N_i} \tag{19}$$

The sensitivity for the active redundancy module with identical units is given by

$$S_{N_i} = S_{R_i}\,[-\,(1 - r_i)^{N_i} \ln\,(1 - r_i)] \tag{20}$$

The sensitivity for the standby modules can be approximately calculated by

$$S_{N_i} \simeq S_{R_i}\,[R_i\,(N_i + 1) - R_i(N_i)] \tag{21}$$

A large sensitivity value means that one may increase the system reliability greatly by increasing the corresponding individual reliability or by increasing the number of units in the corresponding module. Sensitivity, therefore, is a measure of system reliability improvement and provides important information for maximizing the reliability of complex systems.

BASIC ALGORITHM FOR THE PATH ENUMERATION METHOD

The total system reliability and sensitivities can be obtained by the following six-step procedure:

1. Find all minimal paths using the reliability graph; a suitable computer program for finding paths has been developed by Henley and Williams (1973).
2. Find all the required unions of the paths;
3. Give each path union a reliability expression in terms of module reliability;
4. Sum up all the reliability expressions obtained above according to Equation (14);
5. Evaluate the system reliability by substituting numerical values of each module reliability; and,
6. Evaluate the desired sensitivities from Equations (16) to (21).

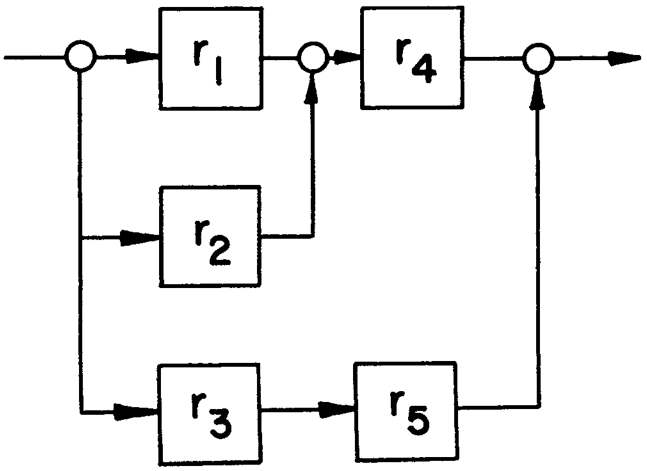

Fig. 3. Reliability graph.

TABLE 1. ALL COMBINATIONS OF THE STATES

State number	r_1	r_2	r_3	r_4	r_5	Giving a path?
	Binary number					
0	0	0	0	0	0	No
1	0	0	0	0	1	No
2	0	0	0	1	0	No
3	0	0	0	1	1	No
4	0	0	1	0	0	No
5	0	0	1	0	1	Yes
6	0	0	1	1	0	No
7	0	0	1	1	1	Yes
8	0	1	0	0	0	No
9	0	1	0	0	1	No
10	0	1	0	1	0	Yes
11	0	1	0	1	1	Yes
12	0	1	1	0	0	No
13	0	1	1	0	1	Yes
14	0	1	1	1	0	Yes
15	0	1	1	1	1	Yes
16	1	0	0	0	0	No
17	1	0	0	0	1	No
18	1	0	0	1	0	Yes
19	1	0	0	1	1	Yes
20	1	0	1	0	0	No
21	1	0	1	0	1	Yes
22	1	0	1	1	0	Yes
23	1	0	1	1	1	Yes
24	1	1	0	0	0	No
25	1	1	0	0	1	No
26	1	1	0	1	0	Yes
27	1	1	0	1	1	Yes
28	1	1	1	0	0	No
29	1	1	1	0	1	Yes
30	1	1	1	1	0	Yes
31	1	1	1	1	1	Yes

TABLE 2. PATHS AND PATH UNIONS

Paths and path unions	Sign	r_1	r_2	r_3	r_4	r_5
		Module				
$P1$	+	1	0	0	1	0
$P2$	+	0	1	0	1	0
$P3$	+	0	0	1	0	1
$P1$ or $P2$	−	1	1	0	1	0
$P1$ or $P3$	−	1	0	1	1	1
$P2$ or $P3$	−	0	1	1	1	1
($P1$ or $P2$) or $P3$	+	1	1	1	1	1

COMPARISON OF THE STATE ENUMERATION ALGORITHM AND THE PATH ENUMERATION METHOD

An alternate method of calculating system reliability is the method of state enumeration. The algorithm for the state enumeration can be briefly stated as follows:

1. Find all the possible combinations of states of the units (up or down);
2. Find each combination which connects input and output; calculate the product of the unreliabilities of the down units and the reliabilities of the up units;
3. Sum the products obtained in Step 2. This gives the system reliability expression.

The algorithm can be easily computerized by using Boolean algebra (Brown, 1971). As an example, consider the reliability graph shown in Figure 3. The algorithm can be understood by referring to Table 1. From the table, the system reliability function is given by

$$R = (1-r_1)(1-r_2)r_3(1-r_4)r_5 + (1-r_1)(1-r_2)r_3r_4r_5 + (1-r_1)r_2(1-r_3)r_4(1-r_5) + (1-r_1)r_2(1-r_3)r_4r_5 + (1-r_1)r_2r_3(1-r_4)r_5 + (1-r_1)r_2r_3r_4(1-r_5) \quad (22)$$
$$+ (1-r_1)r_2r_3r_4r_5 + r_1(1-r_2)(1-r_3)r_4(1-r_5) + r_1(1-r_2)(1-r_3)r_4r_5 + r_1(1-r_2)r_3(1-r_4)r_5 + r_1(1-r_2)r_3r_4(1-r_5) + r_1(1-r_2)r_3r_4r_5 + r_1r_2(1-r_3)r_4(1-r_5) + r_1r_2(1-r_3)r_4r_5 + r_1r_2r_3(1-r_4)r_5 + r_1r_2r_3r_4(1-r_5) + r_1r_2r_3r_4r_5$$

Paths and path unions required in the path enumeration algorithm are listed in Table 2. From Table 2 the system reliability function is easily derived as

$$R = r_1r_4 + r_2r_4 + r_3r_5 - (r_1r_2r_4 + r_1r_3r_4r_5 + r_2r_3r_4r_5) + r_1r_2r_3r_4r_5 \quad (23)$$

Although the two expressions given by Equations (22) and (23) are different in form, it can be easily verified that both are equivalent.

We conclude from the above comparison and from more general consideration that the advantages of the path enumeration method are:

1. Steps to reach the system reliability function are much less if the number of paths is fairly small and is unaffected by the total number of modules in the system;
2. The number of terms in the system reliability function are fewer; and,
3. Any term in the system reliability function contains equal or fewer multipliers.

EXTENSION TO SYSTEM MTBF CALCULATION

A most important parameter of the system reliability is the mean time between failures (MTBF). The sensitivity of the MTBF to the reliability r_{ij} of the jth unit belonging to the ith module and to the number of units in the ith module is given by

$$\frac{\partial \overline{M}}{\partial r_{ij}} = \int_0^\infty \frac{\partial R}{\partial R_i} \frac{\partial R_i}{\partial r_{ij}} dt \quad (25)$$

and

$$\frac{\partial \overline{M}}{\partial N_i} = \int_0^\infty \frac{\partial R}{\partial R_i} \frac{\partial R_i}{\partial N_i} dt \quad (26)$$

Therefore, by the addition of an integration routine to the reliability calculation program described earlier, the system MTBF and its sensitivities can be easily calculated. A computer program containing a numerical integration routine based on Simpson's formula has been developed by Gandhi and Henley (1974). This MTBF calculation routine is more flexible and easier to apply to more complicated systems than the RELCOMP routine of Fleming (1971) which is only applicable to series-parallel configurations.

APPROXIMATIONS TO SYSTEM RELIABILITY

If the number of paths is large (more than 10), the computation time increases rapidly. This is evident from Equation (15) which gives the total number of terms involved in the reliability expression as a function of total number of paths in the system. The same is true for reliability calculation based on cut enumeration. To avoid this difficulty, some lower and upper bounds approximations to system reliability are presented (Messinger and Shooman, 1967; Jensen and Bellmore, 1969; Nelson et al., 1970; Batts, 1971; Locks, 1971).

First reliability approximations based on the path enumeration method are developed. The series given by Equation (11) has the following properties:

$$R \le R_{U_1} = \sum_{i=1}^{M} P_r\{P_i\}$$
$$R \ge R_{L_1} = \sum_{i=1}^{M} P_r\{P_i\} - \sum_{i=1}^{M}\sum_{j>i}^{M} P_r\{P_i \cap P_j\} \quad (27)$$
$$R \le R_{U_2} = \sum_{i=1}^{M} P_r\{P_i\} - \sum_{i=1}^{M}\sum_{j>i}^{M} P_r\{P_i \cap P_j\} + \sum_{i=1}^{M}\sum_{j>i}^{M}\sum_{k>j}^{M} P_r\{P_i \cap P_j \cap P_k\}$$
$$\vdots$$

and

$$R_{U_1} \ge R_{U_2} \ge \ldots \qquad R_{L_1} \le R_{L_2} \le \ldots \quad (28)$$

Therefore, $R_{U_1}, R_{U_2}, \ldots$ can be used as successive upper bounds for R, and $R_{L_1}, R_{L_2}, \ldots$ can be used as lower bounds for R. This is the lower and upper bounds reliability approximation. These approximations are very close when element reliabilities are small. This is also called the *low reliability* region approximation.

In the analogous way, it is possible to develop high reliability region approximation formulae from the cut enumeration method (Jensen and Bellmore, 1969; Messinger and Shooman, 1967; Nelson et al., 1970; Shooman, 1968). From Equation (13)

$$R \ge R_{L_1} = 1 - \sum_{j=1}^{M} P_r\{\overline{C}_j\}$$
$$R \le R_{U_1} = 1 - \sum_{j=1}^{M} P_r\{\overline{C}_j\} + \sum_{j=1}^{M}\sum_{k>j}^{M} P_r\{\overline{C}_j\overline{C}_k\} \quad (29)$$
$$\vdots$$

and

$$R_{U_1} \ge R_{U_2} \ge \ldots$$
$$R_{L_1} \le R_{L_2} \le \ldots$$

From Equation (29) the number of terms in the lower and upper bound computations R_{L_1} and R_{U_1} are M and $M(M+1)/2$, respectively. This is compared to $2^M - 1$ terms obtained by expanding Equation (13).

In the system MTBF calculation both the approximations based on path enumeration and cut enumeration will play an important role because element reliabilities change from high reliability to low reliability regions as time increases from 0 to ∞.

OPTIMAL DESIGN FOR RELIABILITY AND AVAILABILITY OF COMPLEX SYSTEMS

Most of the earlier literature in the area of system reliability optimization considered only redundant series-parallel systems. The reason that this particular system has received so much attention is the separability of the reliability function. For the configuration shown in Figure 2, the system reliability is given by Equation (6). Taking logarithms of both sides of Equation (6) and letting n_j be the total number of identical components in the jth module,

$$\begin{aligned} \ln R(\mathbf{n}, \mathbf{r}) &= \sum_{j=1}^{N} \ln R_j(n_j, r_j) \\ &= \sum_{j=1}^{N} \ln [1 - (1 - r_j)^{n_j}] \end{aligned} \tag{30}$$

Since the natural logarithm is monotonic, maximization of the logarithm is equivalent to maximization of its argument. The form of Equation (30) for reliability maximization is more convenient to use since each term of the sum depends on a single variable. A separable function can be analyzed as a multistage process to which the methods of dynamic programming (Bellman and Dreyfus, 1958; Fyffe et al., 1968; Kettelle, 1962; Liittschwager, 1964; Messinger and Shooman, 1970; Misra, 1971b; Woodhouse, 1962) and discrete maximum principle (Fan et al., 1967; Tillman et al., 1968) are applicable. The problem may also be transformed and solved as an integer programming problem (Ghare and Taylor, 1969; Misra, 1971a; Mizukami, 1968; Tillman and Liittschwager, 1967; Tillman, 1969) or by using the Lagrangian multipliers technique (Barlow et al., 1965; Banerjee and Rajamani, 1973; Everett, 1963; Messinger and Shooman, 1970; Misra, 1972). The concept of dominating sequence (or a family of uncominated allocations) has also been used to solve the problem (Barlow et al., 1965; Black and Proschan, 1959; Kettelle, 1962; Messinger and Shooman, 1970; Proschan and Bray, 1965).

Burton and Howard (1969 and 1971) consider the reliability optimization of a series-parallel-series system. They present a dynamic programming model. The notion of the generalized decomposition operator is used to develop a set of recursive relations. Terano et al. (1970) also consider a series-parallel-series system. They linearize the system at the nominal point and apply dynamic programming to find optimum redundancies. A nonseries-parallel system is treated by Tillman et al. (1970). They use the sequential unconstrained minimization technique to find optimum module reliabilities that maximize the system reliability.

To demonstrate the structure of these optimizations, we will employ the sensitivity function for maximizing the reliability, availability, or profit of a complex system (series-parallel-series or nonseries-parallel) subject to linear or nonlinear constraints. The optimum seeking algorithm which finds the optimum redundancies and preventive maintenance schedules is based on an integer gradient method of Reiter and Rice (1966).

Typically, three types of reliability optimization problems are formulated:

Optimal Allocation of Redundancy

The problem is to find the optimal redundancy for each module (unit reliabilities are specified) so as to maximize the system reliability subject to linear or nonlinear cost constraints and is stated as follows:
Maximize the system reliability

$$R(\mathbf{n}, \mathbf{r}) \tag{31}$$

where $\mathbf{r}$ is specified.
Subject to linear or nonlinear cost constraints

$$\sum_{i=1}^{N} g_{ij}(n_i, r_i) \leq G_j \quad j = 1, 2, \ldots m \tag{32}$$

and non-negative and integer constraints

$$n_i \geq k_i \qquad i = 1, 2, \ldots N \tag{33}$$
$$n_i : \text{integer}$$

where k_i is the minimum number of units in the ith module that must operate for the module to function successfully. This constitutes a nonlinear, integer programming problem.

Optimal Allocation of Unit-Reliability or Unit Maintenance Interval

The problem is to find the optimal unit reliabilities or the optimum maintenance interval for each unit (module redundancies are specified) so as to maximize the system reliability subject to linear or nonlinear cost constraints.

Once the optimal unit reliabilities are known the optimal maintenance interval can be evaluated by the relationship

$$r_i = e^{-T_i/(\mathrm{MTBF})_i} \tag{34}$$

where T_i is the maintenance interval of a unit belonging to the ith module, and $(\mathrm{MTBF})_i$ is the mean time between failure of a unit belonging to the ith module.

The optimization problem can be stated as follows:

Maximize the system reliability

$$R(\mathbf{n}, \mathbf{r}) \tag{35}$$

where $\mathbf{n}$ is specified.

Subject to linear or nonlinear cost constraints

$$\sum_{i=1}^{N} g_{ij}(n_i, r_i) \leq G_j \quad j = 1, 2, \ldots m \tag{36}$$

and non-negative constraints.

$$1 \geq r_i \geq 0 \quad i = 1, 2, \ldots N \tag{37}$$

The problem can be extended to 1-out-of-n active redundancy module with dissimilar units.

The above problem constitutes a nonlinear programming problem.

Mixed Optimal Problem

Maximize the system reliability $R(\mathbf{n}, \mathbf{r})$ with respect to the number of redundant units ($\mathbf{n}$) and unit reliabilities ($\mathbf{r}$) subject to linear or nonlinear cost constraints.

This is a nonlinear mixed integer programming problem.

System Availability and System Profit

System availability can be improved by adding redundancy to the system and/or by performing preventive maintenance on the system according to some prescribed schedule.

In the absence of scheduled preventive maintenance, the system availability A is

$$A(\mathbf{n}) = \frac{\mathrm{MTBF}}{\mathrm{MTBF} + \mathrm{MDT}} = \frac{\int_0^\infty R(\mathbf{n}, t)\,dt}{\int_0^\infty R(\mathbf{n}, t)\,dt + \mathrm{MDT}} \tag{38}$$

Now suppose preventive maintenance is performed on the system every T hours of continuing operation. If the system fails before T hours have elapsed, emergency maintenance is performed at that time. Preventive maintenance is then rescheduled. We assume that the system is as good as new after any type of maintenance, scheduled or emergency, and that the system either operates at full capacity or is down for maintenance.

TABLE 3. RELIABILITY OPTIMIZATION—SYSTEM 1

Module	Optimum number of units in each module									
	Result 1	Result 2	Result 3	Result 4	Result 5	Result 6	Result 7	Result 8	Result 9	Result 10
1	2	2	2	2	1	2	1	1	1	1
2	2	2	1	2	2	1	2	2	2	2
3	2	2	2	1	1	2	1	1	2	1
4	1	1	2	2	2	2	2	2	2	2
5	1	1	1	1	2	1	2	2	1	1
6	1	2	1	1	1	2	1	2	1	1
7	2	1	2	2	1	1	2	1	2	3
8	1	1	1	1	2	1	1	1	1	1
9	2	3	2	3	3	3	2	3	2	2
10	2	1	2	1	1	1	2	1	2	2
Cost utilized	1,286.0	1,294.0	1,283.0	1,288.0	1,299.0	1,291.0	1,290.0	1,298.0	1,298.0	1,293.0
Maximum reliability	0.975982	0.977958	0.978306	0.978900	0.978938	0.980288	0.981123	0.982116	0.982301	0.983708

Let

$f(\mathbf{n}, t)$ = system failure time probability density function

T_E = mean time to perform emergency maintenance on the system

T_S = mean time to perform scheduled maintenance on the system

Thus, mean time between system maintenance is given by

$$\begin{aligned} \text{MTBM} &= T \star R(\mathbf{n}, t) + \int_0^T t\, f(\mathbf{n}, t)\, dt \\ &= \int_0^T R(\mathbf{n}, t)\, dt \end{aligned} \tag{39}$$

Also, assuming that T_E and T_S are independent of $\mathbf{n}$ and T, the mean down time for the system is

$$\begin{aligned} \text{MDT} &= T_E\ (1 - R(\mathbf{n}, T)) + T_S \star R(\mathbf{n}, t) \\ &= T_E - (T_E - T_S) \star R(\mathbf{n}, t) \end{aligned} \tag{40}$$

System availability is then defined as

$$A(\mathbf{n}, T) = \frac{\int_0^T R(\mathbf{n}, t)\, dt}{\int_0^T R(\mathbf{n}, T)\, dt + T_E - (T_E - T_S)\ R(\mathbf{n}, t)} \tag{41}$$

Equation (41) is then substituted into a profit objective function such as

Profit = net income − capital cost − maintenance cost

the net income being directly related to the plant availability. Gandhi and Henley (1974) have published studies of availability optimizations, as have Allen and Pearson (1974).

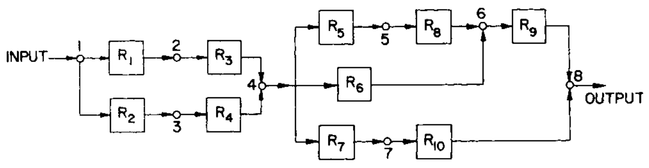

Fig. 4. Reliability graph—System I.

Illustrative Examples

Table 3 shows the results obtained when the system depicted in Figure 4 is manipulated with respect to redundancy to optimize the reliability, subject to an overall cost constraint of 1300. These results were obtained using the Reiter-Rice integer gradient method. The optimum configuration, Result 10, was verified by a Lawler-Bell (1966) partial enumeration technique which always gives a true global optimum.

The point to note here is that there are a large number of near-optimum solutions. This suggests that much of the redundancy optimization literature is not to be trusted because noninteger optimization methods were employed, and the results are rounded off.

FAULT TREE ANALYSIS

It is useful here to distinguish between safety/hazard and reliability analysis even though they form a highly intersecting set.

The modern hazard analysis technique is to construct a fault tree by first identifying a critical top event failure and then enumerating all of the events which can lead to that failure. The concept of fault tree analysis was originated at the Bell Telephone Laboratories in 1961 to perform a safety analysis of the Minuteman Launch Control System. Two recent examples of fault tree applications to chemical process problems are discussed by Lawley (1974) and Stewart and Hensley (1971).

As an example of fault-tree construction, consider the motor-fuse-switch-power supply system of Figure 5 (Fussell, 1974). Suppose now that we wish to calculate the reliability of the system, taking the probability of a motor failure as the top event. (1) By deductive "what-if" logic we can evolve the fault-tree of Figure 5. In this diagram the shield-like symbols represent OR logic, that is, the motor does not operate if there is no current or there is a primary motor failure. In general, complex fault-trees can have AND, OR, INHIBIT and DELAY logic.

To calculate the probability of the motor failing, it is necessary to have failure rate data for the wire, the fuse, the power supply, the switch, and the motor. Given this information, the reliability of the system in terms of the top event failure can be computed. The methods for making these computations involves calculating the cut sets, that is, combination of events leading to failure, and have been reviewed by Fussell (1974).

By comparing the methods developed for fault tree analysis with those used for block diagrams, we see that the

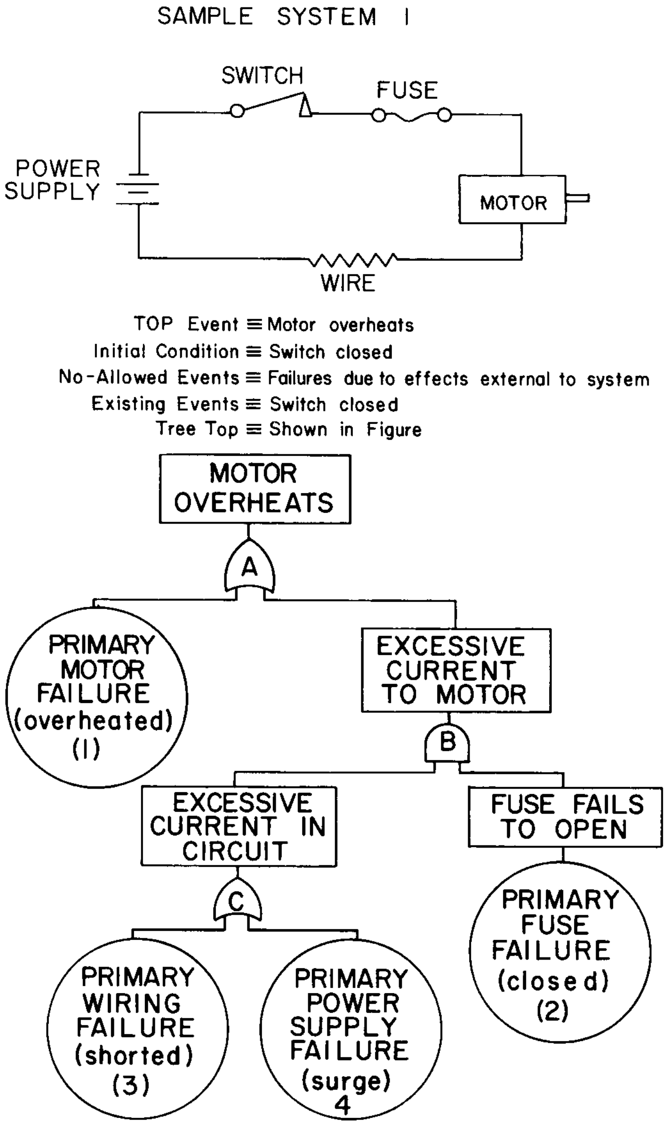

Fig. 5. First fault tree for sample System I.

two fields have developed independently: fault tree people talk in terms of minimal cut sets; reliability graph people in terms of minimal paths. It will now be shown that this dichotomy is unnecessary; a unified approach to the problem is possible.

Let us begin by using Fussell's idea for min cut set generation to develop a new algorithm for min path detection (Inoue and Henley, 1975). We begin by classifying every node in Figure 1 as an OR, or AND node, these being defined, respectively, as nodes with multiple inputs and a single output, and nodes with single inputs and multiple outputs.

The algorithm begins with the node immediately to the left of the output. If the node is an OR node, each input element together with a predecessor node is used as an entry in a new row of a list table. If the node is an AND node, each input element is used as an entry in the first row of the list table. With reference to Figure 1, since the first node d is an OR node, we have

$$R_4\ b$$
$$R_3\ c$$

The basic idea of the algorithm is to replace each node by its input element with its predecessor node until the input node appears in every row. Continuing thusly, since b is an AND node and c is an OR node,

$$R_4\ R_1\ a$$
$$R_3\ R_2\ b$$
$$R_3\ R_5\ a$$

Finally, we have

$$R_4\ R_1\ a$$
$$R_3\ R_2\ R_1\ a$$
$$R_3\ R_5\ a$$

Each row now represents a minimal path. To obtain the cut sets we begin by replacing all events by their corresponding duals, that is, the nonoccurrence of the original basic events, and we replace the OR gates by AND gates, and vice versa.

The procedure to final cut set generation for Figure 1 is shown below

$$\begin{array}{c} R_4R_3 \\ b\ c \end{array}$$

$$\begin{array}{cc} R_1R_3 & R_4R_2R_5 \\ a\ c & b\ b\ a \\ \downarrow & \downarrow \\ R_1R_5 & R_1R_5 \\ a\ a & a\ a \end{array}$$

The algorithm detects all the min cuts.

Another interesting method for obtaining minimum cut sets from reliability graphs utilizes the concept that if we multiply, from the left-hand side, a distar matrix **D** of a directed graph by a matrix **Q** which includes all the sets of two-partition of the node set into a subset that includes the input node, and the other that includes the output node, then we have a cut set matrix **C** which includes all the cut sets.

Applying this idea to the graph of Figure 1

$$\mathbf{C} = \mathbf{Q} \times \mathbf{D} = \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \end{array} \begin{array}{cc} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix} & \begin{array}{c} a \\ b \\ c \\ d \end{array} \end{array} = \begin{array}{cc} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} & \begin{array}{c} a \\ b \\ c \\ d \end{array} \end{array}$$

Since only the out cut sets are useful, it is seen that matrix **C** correctly gives the cut sets R_1R_5, R_1R_3, $R_2R_4R_5$, R_3R_4.

A third and very simple algorithm for obtaining minimal cuts can best be demonstrated by inspection of the path matrix for the reliability graph of Figure 1.

		R_1	R_2	R_3	R_4	R_5
Path	1	1	1	1	0	0
	2	1	0	0	1	0
	3	0	0	1	0	1

For path 1 to fail, the element R_1, R_2, or R_3 should fail. For path 2 to fail, R_1 or R_4 should fail, etc. For all paths to fail

$$(R_1 + R_2 + R_3)(R_1 + R_4)(R_3 + R_5)$$

must be true. By expanding the equation and absorbing terms using the formula

$$R_iR_j + R_j = R_j$$

we obtain

$$R_1R_3 + R_3R_4 + R_1R_5 + R_2R_4R_5$$

The ability to convert min paths to min cuts permits the computerization of fault tree construction, given a block diagram. A program for doing this is being developed at Houston (Caceres, 1974).

NOTATION

A = system availability
C_j = jth minimal cut
$\overline{C_j}$ = complement of the event C_j
f = failure density function
F = system unreliability
g_{ij} = contribution of ith module to jth constraint
G_j = upper limit on jth constraint
k_i = minimum number of units in the ith module that must function for the module to operate successfully
MDT = system mean downtime
MTBF = system mean time between failure
N = total number of modules in the system
N_i = number of units in the ith module
P_i = ith minimal path
R = system reliability
R_i = ith module reliability
r_i = reliability of a unit of the ith module
r_{ij} = reliability of jth unit of the ith module
R_L = lower bound approximation for R
R_U = upper bound approximation for R
S_{Ri} = sensitivity of system reliability to the ith module reliability
S_{rij} = sensitivity of system reliability to the reliability of the jth unit of the ith module
t = variable of integration
T_E = mean time for emergency maintenance on the system
T_i = maintenance interval of a unit of the ith module
T_S = mean time for scheduled maintenance on the system
Z = total number of terms
z = hazard rate

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Continuous Production of Polystyrene in a Tubular Reactor: Part I

An experimental analysis of the bulk polymerization of styrene initiated by azobisisobutyronitrile was carried out in a tubular reactor.

The experiments performed in a 2.362-cm I.D., 6-meter long jacketed reactor showed that: (1) it was feasible for a tubular reactor to produce acceptable quality polystyrene for industrial purposes; (2) the quality of the polymer product was reproducible at any time; (3) it was technically feasible for the tubular reactor to replace the stirred-batch kettles as the preliminary stage in the polystyrene manufacturing process, and (4) radial temperature gradients were not a problem for reactor operation.

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SCOPE

Over 50% of the polystyrene manufactured comes from bulk polymerization processes (Bishop, 1971). The major manufacturing process is usually a two-stage process (Boundy et al., 1970), the first stage (stirred-batch kettles) takes the conversion of bulk monomer to 30 to 40% completion. Upon reaching this level of conversion, the greatly increased viscosity of the polymer solution becomes a serious hindrance to the mixing of the polymer syrup resulting in poor control of temperature and, hence, the molecular weight distribution. At the same time there is an increased energy requirement for the mixer. This impairs the economics of the process. Therefore, the polymer syrup is transferred to processing equipment such as screw extruders, plate and frame presses, or temperature controlled towers, which are more suited to handle the viscous mixture.